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Trapezoidal sum for $\int_a^b f(x) dx$
First order Taylor polynomial for $f(x)$
First order Taylor polynomial for $f(x)$ with $x_0 = 0$
Taylor polynomial for $\cos(x)$ with $x_0 = 0$
Area under $f(x)$
Volume of a cube around $f(x)$