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**11 The Application of Computer Models**

Computer models are everywhere in use. Whether it is the manufacturing industry, the electricity generating utility, the airways, the railway, financial institutions, the oil and chemical industry, hospitals or the government, they all use the output from computer models as input to their decision-making.

- At the base of the decisions are problems such as the:
- Prediction of economic growth.
  - Assessment of health, technical and regulatory risks.
  - Reliable supply of electricity and drinking water.
  - Scientific operation of waste water treatment plants.
  - Maintenance of oil reservoirs for sale and optimum production.
  - Expansion of transport plant and infrastructure strategies for infectious diseases.
  - Completion of safety requirements for potentially hazardous industries.
  - Choice of sustainable harvesting quotas for natural resources like forests and fish populations.
  - Adequate appreciation of the human influence on global climate.
  - Sustainable management of industrial portfolios.
  - Optimum scheduling of aircrafts and airways.

In order to solve these problems a number of questions are to be asked. Computer models are applied to find the answers that are then used in decision-making. Applying a computer model is the only way of finding the answers since observations, tests or experiments are often out of the question. This is particularly so if the systems are comprised with potentially hazardous events or with predictions for the future.

A computer model is an encoded, simplified mathematical abstraction of reality. It is limited because only those influential, measurable quantities are included that are